MATH 1110: QUICK REVIEW

the derivative of $\sin(x)$ is: $\cos x$

1. By using one of the trig identities listed below, use the derivative of sine and derivative rules to compute the derivative of cosine.

$$\cos(x) = \sqrt{1-\sin^2(x)} \quad \text{(a rearrangement of } \sin^2(x)+\cos^2(x)=1\text{)}$$
$$\cos(x) = \sin(x+\tfrac{\pi}{2})$$
$$\cos(x) = 1-\tfrac{1}{2}\sin^2(\tfrac{x}{2})$$

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the derivative of $\cos(x)$ is:

2. By writing each trig function in terms of sine & cosine and then using derivative rules (e.g. Chain Rule, Product Rule), compute the derivative of each of the following:
   (a) $\tan(x)$ $= \dfrac{\sin x}{\cos x}$, so $\dfrac{d}{dx}\tan x = \dfrac{\cos x(\cos x) - \sin x(-\sin x)}{(\cos x)^2}$
   $= \dfrac{1}{(\cos x)^2} = \boxed{(\sec x)^2}$
   (b) $\sec(x)$
   (c) $\csc(x)$ $= \dfrac{1}{\sin x}$, so $\dfrac{d}{dx}\csc x = \dfrac{d}{dx}\left((\sin x)^{-1}\right) = -1\cdot(\sin x)^{-2}\cos x$
   $= \dfrac{-\cos x}{(\sin x)^2}$
   $= \boxed{-\csc x \cot x}$
   (d) $\cot(x)$

Date: Tuesday, 2 October 2018.
Instructor: Zach Norwood.

(i) $\frac{d}{dx}\cos x = \frac{d}{dx}\sqrt{1-\sin^2 x} = \frac{1}{2}\left(1-\sin^2 x\right)^{-1/2}\left(-2\sin x\cos x\right)$

$$= \frac{-2\sin x\cos x}{2\sqrt{1-\sin^2 x}} = \frac{-2\sin x\cancel{\cos x}}{2\cancel{\cos x}}$$

$$= \boxed{-\sin x}$$

(ii) $\frac{d}{dx}\cos x = \frac{d}{dx}\sin\left(x+\frac{\pi}{2}\right) = \cos\left(x+\frac{\pi}{2}\right)\cdot 1$

$$= \cos\left(-\left(x+\frac{\pi}{2}\right)\right) = \cos\left(-x-\frac{\pi}{2}\right)$$

cosine is even

$$= \sin(-x)$$

$\sin\theta = \cos\left(\theta-\frac{\pi}{2}\right)$

sine is odd $\quad = -\sin x$ ✓

(iii) $\frac{d}{dx}\cos x = \frac{d}{dx}\left(1-\frac{1}{2}\sin^2\left(\frac{x}{2}\right)\right) = -\sin\left(\frac{x}{2}\right)\cdot\cos\left(\frac{x}{2}\right)\cdot\frac{1}{2}$

$$= -\frac{1}{2}\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) = -\sin\left(2\cdot\frac{x}{2}\right) = -\sin x \checkmark$$

$\sin(2\theta) = 2\sin\theta\cos\theta$

3. Compute each of the following derivatives:
   (e) $\tan(x^2)$ and $(\tan(x))^2$

   $\sec^2(x^2)\cdot 2x$ $\qquad$ $2\tan x \sec^2 x$

   (f) $e^{\cos x}$

   $e^{\cos x}(-\sin x)$

   (g) $\cos^2(\sin(x)) + \sin^2(\sin(x))$ $= 1$

   deriv $= 0$

   (h) $e^x + 2x + \sin x - 6$

   $e^x + 2 + \cos x$

   (i) $e^{\sec x} + 2\sec x + \sin(\sec x) - 6$

   $\left(e^{\sec x} + 2 + \underbrace{\cos(\sec x)}_{=x}\right)(\sec x \tan x)$

   (j) e^{e^x}

   $e^{e^x}\cdot e^x$

   (k) $e^{e^{e^{e^{\cos x}}}}$ $\rightsquigarrow$ $e^{e^{e^{e^{\cos x}}}} \cdot e^{e^{e^{\cos x}}} \cdot e^{e^{\cos x}} \cdot e^{\cos x} \cdot (-\sin x)$