FORCING EXERCISES – DAY 1

Exercise 0. Make sure you know how to do the prep material problems, especially #18.

Exercise 1. Let κ, λ, and μ be cardinals.
(a) Show that $(\kappa^\lambda)^\mu = \kappa^{\lambda\cdot\mu}$.
(b) Show that $\kappa^\kappa = 2^\kappa$.

Exercise 2. Let A be an infinite set of size κ. Show that the set of all bijections from A to A has size 2^κ.

Exercise 3. Let κ and λ be cardinals with $\lambda \leq \kappa$ and κ infinite. We write $[\kappa]^\lambda$ for the collection of all subsets of κ of size λ. Show that the cardinality of $[\kappa]^\lambda$ is κ^λ.

Exercise 4. Show that the union of a set of cardinals is a cardinal.

Exercise 5. Suppose that $\langle \alpha_i : i < \lambda \rangle$ is an increasing sequence of ordinals cofinal in some cardinal κ. Show that $\mathrm{cf}(\kappa) = \mathrm{cf}(\lambda)$.

Exercise 6. Let κ be an infinite cardinal and $\prec$ a wellordering on κ. Show that there is a subset of κ of size κ on which $\prec$ and the usual ordering $<$ agree.

Exercise 7. Show in ZF that the Axiom of Choice is equivalent to the statement that, for every ordinal α, the powerset $\mathscr{P}(\alpha)$ can be wellordered.

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